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The approximate temperatures within a flat-plate solar collector under transient conditions

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Abstract—Some of the heat resistances of a conventional flat-plate solar collector are smaller than others. Using this property, analytic approximations of the temperatures within the collector at a moment $t = 0$, are derived. They are expressed as functions of the time-varying behaviour of the insolation, the ambient temperature and the fluid outlet and inlet temperature. In particular, the fast-varying temperature of the fluid near the collector inlet has been shown. Some applications are mentioned, in which coarser approximations of the temperatures are possible. A simple, one-capacity differential equation describes the fluid temperature, in that case.

1. INTRODUCTION

OPERATING a solar collector during varying weather conditions, the collector temperatures are influenced by thermal heat capacities. In calculations of the collector performance, or when considering a test-method for collectors under transient weather conditions, the difference in the heat stored capacitively between the moment of the beginning, $t = 0$, and the end, $t = t_e$, of the measurements, may play a role.

Insight in the importance of this effect is obtained by solving a set of differential equations, representing the transient behaviour of the collector. Usually, the method of solution requires the temperatures within the collector to be known as boundary conditions at $t = 0$, or $t = t_e$, respectively.

Measurements of these temperatures inside the collector are generally impossible. In this paper, it will be derived that for the conventional flat-plate solar collector, the various temperatures in the collector at $t = 0$ can be approximated using only the measurements of the insolation, the ambient temperature and the fluid outlet temperature during a certain time. The derivation takes advantage of the circumstance that some of the heat resistances in the collector are smaller than others. The approximation is particularly important for a test-method, developed in our laboratory, where the collector is placed outside and operates essentially under actual sunlight, and consequently unpredictable weather conditions. In this case, it is impossible to start the calculations from a stationary situation or to consider a strictly periodical behaviour of the collector as mentioned in the literature on dynamic laboratory experiments [1, 2].

As a result of the calculations, it is found that in many applications the thermal behaviour can be approximately described by one capacity, consisting of the sum of the fluid and absorber capacity.

2. THE MATHEMATICAL REPRESENTATION OF A FLAT-PLATE COLLECTOR

The mathematical model describing the properties of a flat-plate collector sketched in Fig. 1 has to be as simple as possible. On the other hand, no essential properties of the collector should be neglected beforehand. A reasonable compromise between the two opposite requirements is found by a four-node model representing the cover plate, the absorber, the fluid and the insulation, respectively (see Fig. 2). The effects caused by the circulating flow within the collector are included by the definition that the temperatures of the collector depend on the coordinate in the direction of the fluid flow. Applying heat balances in a suitable way, the following set of partial differential equations can be

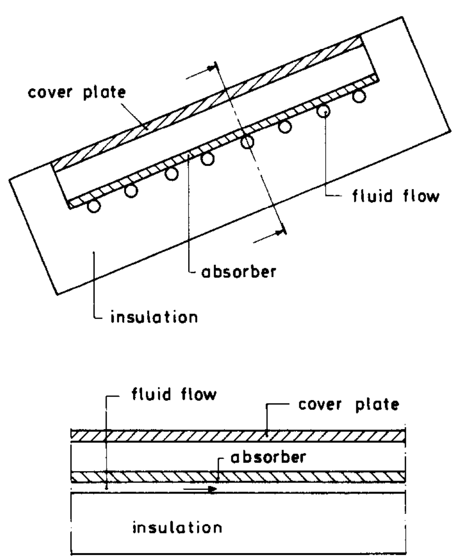

FIG. 1. A sketch of the flat-plate solar collector (not to scale).

NOMENCLATURE

T, θ	the temperature of a collector element [K, –]
ΔT	characteristic quantity of temperature [K]
$\hat{T}$	the Fourier transform of T [K s]
E, ε	the insolation [W m^{-2}, –]
$\hat{E}$	the Fourier transform of E [W m^{-2} s]
C	heat capacity per m^2 [J K^{-1} m^{-2}]
γ	ratio of $C_p + C_f$ and C_f [–]
R	thermal resistance per m^2 [W^{-1} K m^2]
R_L	total heat loss resistance per m^2 [W^{-1} K m^2]
v	ratio of two heat resistances [–]
t, τ	coordinate of time [s, –]
t_0	characteristic unit of time [s]
κ	ratio of time constants [–]
κ_d	dimensionless transit time of the fluid through the collector [–]
t_e, τ_e	time of the measurements [s, –]
ω	Fourier variable [s^{-1}]
x, ξ	coordinate in the flow direction [m, –]
L	length of the collector in the flow direction [m]
β	absorption–transmission product [–]
u	velocity of the fluid flow [m s^{-1}]
c	specific heat of water [J kg^{-1} K^{-1}]
$\dot{m}$	mass flow rate per m^2 [kg s^{-1} m^{-2}]
$Z(\omega)$	generalized expression of the useful energy [J m^{-2}]
$X(\omega)$	the Fourier transform of the insolation [J m^{-2}]
$Y(\omega)$	the Fourier transform of the difference of the fluid inlet temperature and the ambient temperature [K s].

Subscripts

a	ambient
g	cover plate
p	absorber
i	insulation
f	fluid.

derived

$$C_g \frac{\partial T_g}{\partial t}(x,t) = \frac{T_a(t) - T_g(x,t)}{R_{ag}} + \frac{T_p(x,t) - T_g(x,t)}{R_{pg}} \quad (1)$$

$$C_p \frac{\partial T_p}{\partial t}(x,t) = \frac{T_g(x,t) - T_p(x,t)}{R_{pg}} + \frac{T_i(x,t) - T_p(x,t)}{R_{ip}} + \frac{T_f(x,t) - T_p(x,t)}{R_{pf}} + \beta E(t) \quad (2)$$

$$C_f \left\{ \frac{\partial T_f}{\partial t}(x,t) + u \frac{\partial T_f(x,t)}{\partial x} \right\} = \frac{T_p(x,t) - T_f(x,t)}{R_{pf}} \quad (3)$$

$$C_i \frac{\partial T_i}{\partial t}(x,t) = \frac{T_p(x,t) - T_i(x,t)}{R_{ip}} + \frac{T_a(t) - T_i(x,t)}{R_{ip}}. \quad (4)$$

This model includes several approximations. Among others, we neglect the radiation absorbed by the cover plate and the heat losses from the edges of the collector. The heat flows between the parts of the collector are normal to the direction of the fluid flow, while the heat flow in the fluid flow direction is maintained by the circulating fluid only. The influence of the collector temperatures and the wind velocity on the collector parameters, such as heat resistances and heat

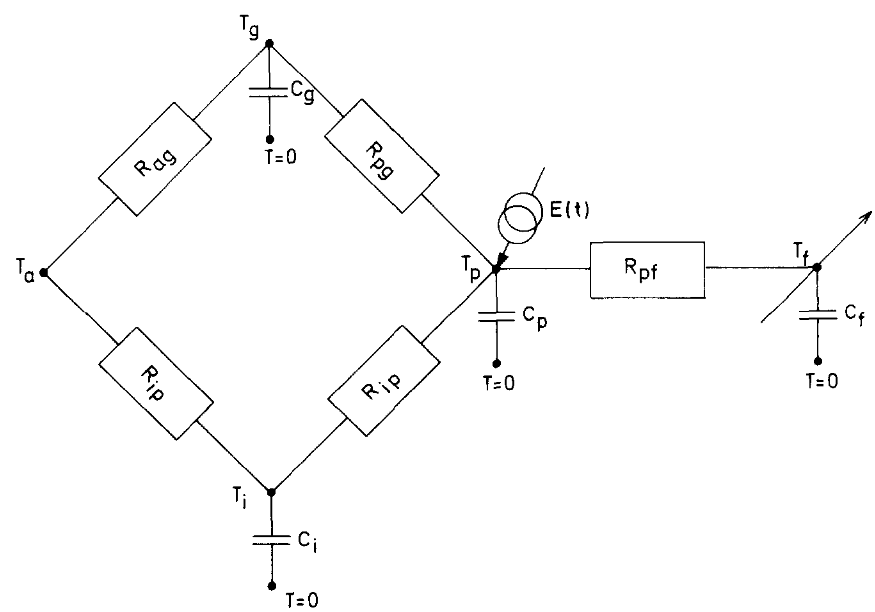

FIG. 2. The resistance analogue illustrating the collector model.

capacities, is neglected. Hence these parameters as well as the velocity of the fluid flow in the collector, are assumed to be constant.

In order to solve the set of differential equations (1) to (4), certain initial conditions are required. Formally, the definition of the collector temperatures at a certain time $t = 0$, say

$$T_g(x,0),\ T_p(x,0),\ T_f(x,0),\ T_i(x,0),\quad 0 \leqslant x \leqslant L, \quad (5)$$

suffices. However, the measurement of a collector temperature within the collector being generally impossible, the conditions (5) are rather unrealistic. In section 6, it will be shown, that appropriate expressions for the temperatures (5) can be obtained using measurements of the insolation, the ambient temperature and the fluid outlet temperature during a finite period after $t = 0$.

It appears convenient to represent the set (1)–(4) in a dimensionless form. For that reason, the following dimensionless quantities are introduced

$$\tau = t/t_0,\ \xi = x/L,\ \theta(\xi,\tau) = T(x,t)/\Delta T,$$
$$\varepsilon(\tau) = \gamma R_L E(t)/\Delta T, \quad (6)$$

where the total heat loss resistance R_L and the fluid flow rate $\dot{m}c$ are given by

$$R_L = 2R_{ip}(R_{ag}+R_{pg})/(2R_{ip}+R_{ag}+R_{pg}),$$
$$\dot{m}c = C_f u/L, \quad (7)$$

and, furthermore

$$t_0 = R_L(C_p+C_f),\ \gamma = (C_p+C_f)/C_f. \quad (8)$$

It will be clear that the characteristic quantities of time, length and temperature are, respectively, given by t_0, the length of the fluid flow in the collector, L, and ΔT. Usually, the last quantity is defined by the difference of two fixed temperatures in the system.

Defining the dimensionless parameters

$$\kappa_g = C_g R_{ag}/t_0,\ \kappa_p = C_p R_{pf}/t_0,$$
$$\kappa_f = C_f R_{pf}/t_0,\ \kappa_i = C_i R_{ip}/t_0,$$
$$\kappa_d = C_f/(t_0 \dot{m}c) \quad (9)$$

and

$$\nu_g = R_{ag}/R_{pg},\ \nu_p = R_{pf}/R_{pg},\ \nu_i = R_{pf}/R_{ip}, \quad (10)$$

the equations are transformed into the equivalent form

$$\kappa_g \frac{\partial \theta_g}{\partial \tau} = \theta_a - \theta_g + \nu_g(\theta_p - \theta_g) \quad (1a)$$

$$\kappa_p \frac{\partial \theta_p}{\partial \tau} = \nu_p(\theta_g - \theta_p) + \nu_i(\theta_i - \theta_p) + \theta_f - \theta_p + \beta\varepsilon\kappa_f \quad (2a)$$

$$\kappa_f \left\{ \frac{\partial \theta_f}{\partial \tau} + \frac{1}{\kappa_d}\frac{\partial \theta_f}{\partial \xi} \right\} = \theta_p - \theta_f \quad (3a)$$

$$\kappa_i \frac{\partial \theta_i}{\partial \tau} = (\theta_p - \theta_i) + (\theta_a - \theta_i). \quad (4a)$$

In order to get an idea of the magnitude of the various quantities, some typical values of some collector parameters will be given. They may be found in textbooks (e.g. [3]). Here we take the values of a properly designed conventional flat-plate collector given in a previous paper of the author [4]. The magnitudes are

$$R_{pg} = 0.25\ \text{K W}^{-1}\ \text{m}^2,\quad R_{ag} = 0.029\ \text{K W}^{-1}\ \text{m}^2,$$
$$R_{pf} = 0.0022\ \text{K W}^{-1}\ \text{m}^2,\quad R_{ip} = 1.25\ \text{K W}^{-1}\ \text{m}^2,$$
$$C_g = 6000\ \text{J K}^{-1}\ \text{m}^{-2},\quad C_p = 3600\ \text{J K}^{-1}\ \text{m}^{-2},$$
$$C_f = 6720\ \text{J K}^{-1}\ \text{m}^{-2},\quad C_i = 1680\ \text{J K}^{-1}\ \text{m}^{-2}, \quad (11)$$

while a fluid flow rate of $\dot{m}c$ equal to 150 W $\text{K}^{-1}\ \text{m}^{-2}$ has been considered. For the various quantities, they lead to

$$\nu_g = 0.11,\ \nu_p = 0.009,\ \nu_i = 0.002,\ \gamma = 1.5,$$
$$\kappa_g = 0.07,\ \kappa_p = 0.003,\ \kappa_f = 0.006,\ \kappa_i = 0.8,$$
$$\kappa_d = 0.017. \quad (12)$$

The data of the collector show that the resistances R_{ag} and R_{pf} are small with respect to R_{pg}, at the other hand, R_{pg} is smaller than R_{ip}. The mutual difference between the capacities being less, the capacities, with the exception perhaps of C_i, are roughly speaking of the same order of magnitude. For the corresponding quantities, there results:

$$\nu_g,\ \nu_p,\ \nu_i,\ \kappa_g,\ \kappa_p,\ \kappa_f \ll 1. \quad (13)$$

Although these conclusions are valid for a particular collector, they are representative for a wide range of flat-plate collectors. Hence, they will be used to derive approximate expressions of the temperatures inside the collector, (equation (5)). In the following, the quantities ν_g, ν_p, ν_i, κ_g, κ_p and κ_f will be neglected with respect to one.

3. A DISCUSSION LEADING TO AN APPROXIMATE MODEL

In order to discuss an approximate solution of our original problem described in (1)–(4), it seems attractive to consider an idealized model, in which each of the quantities R_{ag}, R_{pf} and $1/R_{ip}$ has been put equal to the limiting value of zero. This model represents a collector with an excellent heat transfer between the absorber and the fluid, ideally insulated at the back side of the collector and situated in a windy climate.

In this limiting case the values of the quantities on the left-hand side of (13) are taken equal to zero in the corresponding equations (1a)–(4a). It can be seen that when doing so, the role of ν_i depending on R_{ip} is different from that of the quantities depending on R_{pf} or R_{ag}, e.g. κ_p and κ_g. Consider for instance equation (2a),

$$\kappa_p \frac{\partial \theta_p}{\partial \tau} = \nu_p(\theta_g - \theta_p) + \nu_i(\theta_i - \theta_p) + (\theta_f - \theta_p) + \beta\varepsilon\kappa_f. \quad (2a)$$

Contrary to the case where κ_p equals zero, leading to a reduction of the original differential equation into an algebraic one, the character of the differential equation remains the same if v_i equals zero. The latter conclusion implies, that a first-order approximation of the solution with respect to v_i is found by solving the set of equations (1a–4a) with v_i put equal to zero.

A more accurate solution could be obtained by approximating each collector temperature by the first two terms of a power expansion with respect to v_i, thus

$$\theta(\xi, \tau; v_i) = \theta_0(\xi, \tau) + v_i \theta_1(\xi, \tau) + \cdots. \tag{14}$$

Substituting expressions of the type (14) for the temperatures θ_g, θ_p, θ_f and θ_i, respectively, into the set of equations (1a–4a) leads to sets of differential equations, for the approximate functions $\theta_0, \theta_1, \ldots$ of (14), which can be solved successively.

From now on we draw our attention to the approximate solution with respect to the quantities depending on R_{ag} or R_{pf}. We consider the set of equations (1a–3a), where in (2a) v_i is taken equal to zero.

Eliminating θ_g and θ_p, respectively, from (1a–3a), this set transforms into the equivalent linear differential equation of the third order,

$$\kappa_g\left[\kappa_p\left\{\frac{\partial^3\theta_f}{\partial\tau^3} + \frac{1}{\kappa_d}\frac{\partial^3\theta_f}{\partial\xi\partial\tau^2}\right\} + \gamma\frac{\partial^2\theta_f}{\partial\tau^2} + \frac{1}{\kappa_d}\frac{\partial^2\theta_f}{\partial\xi\partial\tau} - \beta\frac{\partial\varepsilon(\tau)}{\partial\tau}\right] + \kappa_p\left[\frac{\partial^2\theta_f}{\partial\tau^2} + \frac{1}{\kappa_d}\frac{\partial^2\theta_f}{\partial\xi\partial\tau}\right] + \gamma\left(\frac{\partial\theta_f}{\partial\tau}\right) + \frac{1}{\kappa_d}\frac{\partial\theta_f}{\partial\xi} + \gamma\theta_f = \gamma\theta_a(\tau) + \beta\varepsilon(\tau). \tag{15}$$

As already mentioned, taking κ_g and κ_p equal to zero in (15), the differential problem rigorously changes as a result of the lowering of the order. In that case it will be clear that the solution of the approximate problem is fully determined by a reduced number of the boundary and initial conditions. Generally the remaining boundary conditions are not satisfied by the approximate solution. As will be seen later on, especially near some of these boundaries, the solution of the original problem considerably differs from the approximate one.

4. AN APPROXIMATE SOLUTION FOR SMALL VALUES OF κ_p

In order to prevent complicated mathematical constructions, we start to investigate the properties of a simpler equation then (15), namely (15), without the first term (with κ_g as a factor), given as

$$\kappa_p\left\{\frac{\partial^2\theta_f}{\partial\tau^2} + \frac{1}{\kappa_d}\frac{\partial^2\theta_f}{\partial\xi\partial\tau}\right\} + \gamma\frac{\partial\theta_f}{\theta\tau} + \frac{1}{\kappa_d}\frac{\partial\theta_f}{\partial\xi} + \gamma\theta_f = h(\tau) \tag{16}$$

in which

$$h(\tau) = \gamma\theta_a(\tau) + \beta\varepsilon(\tau). \tag{17}$$

In this section an approximate solution of (16) for small values of κ_p will be discussed. The reasoning to solve differential problems with a small parameter is in fact standard, and the calculations are straightforward in the case of a simple ordinary differential equation.

Our case is a partial differential equation in the two independent variables ξ and τ. Instead of constants, the general solution has arbitrary functions to be determined from the boundary conditions, in this case. The determination of these functions is essentially more difficult and certainly non-trivial.

For this reason, and since the method of solution is rather unusual in solar collector research, the solution will be given in more detail.

A solution of (16) is composed of the sum of the solution of the corresponding homogeneous equation

$$\kappa_p\left\{\frac{\partial^2\theta_f}{\partial\tau^2} + \frac{1}{\kappa_d}\frac{\partial^2\theta_f}{\partial\xi\partial\tau}\right\} + \gamma\frac{\partial\theta_f}{\partial\tau} + \frac{1}{\kappa_d}\frac{\partial\theta_f}{\partial\xi} + \gamma\theta_f = 0 \tag{18}$$

and a particular solution of

$$\kappa_p\left\{\frac{\partial^2\theta_f}{\partial\tau^2}\right\} + \gamma\frac{\partial\theta_f}{\partial\tau} + \gamma\theta_f = h(\tau). \tag{19}$$

The right-hand side of (19) depends on the time τ only. Applying the method of variation of constants in the usual way [5], yields a particular solution of (19), namely

$$\psi_f(\tau) = \frac{1}{\gamma}\left[\int_0^\tau e^{-(\tau-v)}\,h(v)\,dv - \int_0^\tau e^{-(\tau-v)/\kappa_p}\,h(v)\,dv\right]. \tag{20}$$

Observing that as a result of $\kappa_p \ll 1$, the integrand of the second integral in (20) decreases very fast to zero for $v < \tau$, (20) can be approximated by

$$\psi_f(\tau) = \frac{1}{\gamma}\int_0^\tau e^{-(\tau-v)}\,h(v)\,dv. \tag{21}$$

For small values of κ_p, an approximate solution of (18) can be obtained using a method of Wentzel, Kramers and Brillouin (the so-called WKB-approximation), [5].

We try a solution of the type

$$\theta_f(\xi, \tau; \kappa_p) = e^{a(\xi,\tau)/\kappa_p}\{a_0(\xi, \tau) + \kappa_p a_1(\xi, \tau)\ldots\}. \tag{22}$$

The unknown functions a, a_0, a_1, . . . have to be determined from relations obtained by the substitution of (22) for θ_f and corresponding expressions for its derivatives in (18) and comparing the coefficients of the same order of κ_p. Comparing the coefficients of κ_p^{-1}, the first-order partial differential equation for a (ξ, τ) is found to be

$$\frac{\partial a}{\partial\xi} = -\kappa_d\frac{\frac{\partial a}{\partial\tau}\left(\frac{\partial a}{\partial\tau} + \gamma\right)}{\frac{\partial a}{\partial\tau} + 1}. \tag{23}$$

This equation has solutions, [6], given by

$$a(\xi, \tau) = B + A\{\tau - \kappa_d(1 + \gamma_0)\xi\}, \tag{24}$$

in which A and B represent constants and

$$\gamma_0 = \frac{\gamma - 1}{A+1}. \quad (25)$$

Next, comparing the coefficients of $(\kappa_p)^0$ and using the expression for $a(\xi, \tau)$, from (24), a relation for $a_0(\xi, \tau)$ can be derived, namely

$$P\frac{\partial a_0}{\partial \tau} + Q\frac{\partial a_0}{\partial \xi} = Ra_0, \quad (26)$$

in which

$$P = \frac{(A+1)^2 + (\gamma - 1)}{A+1}, \quad Q = \frac{A+1}{\kappa_d}, \quad R = -\gamma. \quad (27)$$

The integration of the first-order partial differential equation with constant coefficients (26), based on the method of characteristics, results in the following complete solution, [6],

$$a_0(\xi, \tau) = f\left(\tau - \frac{P}{Q}\xi\right) e^{(R/Q)\xi}, \quad (28)$$

f being any function, in a particular case to be determined from the boundary or initial conditions and

$$\frac{P}{Q} = \kappa_d\left[1 + \frac{\gamma - 1}{(A+1)^2}\right], \quad \frac{R}{Q} = \frac{-\gamma\kappa_d}{A+1}. \quad (29)$$

At this point we remark that any function of the type

$$\theta_f(\xi, \tau) = e^{a(\xi, \tau)/\kappa_p}\, a_0(\xi, \tau), \quad (30)$$

in which $a(\xi, \tau)$ and $a_0(\xi, \tau)$ are defined by (24) and (28) respectively, represents an approximate solution of the homogeneous equation (18). This equation being a homogeneous linear differential equation of the second-order with constant coefficients, the general solution of (18) can be written as the sum of two solutions of the type (28). Hence, the approximate solution of (16) is found to be

$$\theta_f(\xi, \tau) = \psi_f(\tau) + e^{(a(\xi, \tau))_1/\kappa_p} f\left(\tau - \frac{P_1}{Q_1}\xi\right) \times e^{(R_1/Q_1)\xi} + e^{(a(\xi, \tau))_2/\kappa_p} g\left(\tau - \frac{P_2}{Q_2}\xi\right) e^{(R_2/Q_2)\xi}. \quad (31)$$

In this approximation the terms with $a_1, a_2, \ldots$ in (22) have been neglected. The two different values of P/Q and R/Q can be obtained from two different choices of A in (29). The problem now is to choose these values of A and the functions f and g in such a way that the boundary conditions are satisfied. In other words the approximate solutions of $\theta_f(0, \tau)$ and $\theta_f(1, \tau)$ obtained from (31) have to be equal to the measured values of the inlet and outlet temperature, respectively. A direct and straightforward calculation of the functions f and g from the boundary conditions, avoiding complicated mathematical constructions, is possible in the case where f and g have the same arguments. From (29), this appears to be the case if A_1 and A_2 are related by

$$A_2 = -2 - A_1 \quad (32)$$

Determining the solution in a suitable way using the assumption (32), it appears that the final solution still depends on the constant A_1, to be chosen arbitrary. However, giving κ_p the limiting value of zero in this expression, it is found that the solution of θ_f remains only finite if A_1 is taken equal to zero. A detailed description of the calculations is not given here.

Giving A_1 and A_2 the value of 0 and -2, respectively, the temperature θ_f, (31), can be written as

$$\theta_f(\xi, \tau) = \psi_f(\tau) + f(\tau - \kappa_d\gamma\xi)\, e^{-\gamma\kappa_d\xi} + g(\tau - \kappa_d\gamma\xi)\, e^{-2(\tau - \kappa_d(2-\gamma)\xi)/\kappa_p}\, e^{\gamma\kappa_d\xi}. \quad (33)$$

The functions f and g are determined from the measurements of the inlet and outlet temperatures as functions of τ and using (33) for $\xi = 0$ and $\xi = 1$, respectively,

$$\theta_f(0, \tau) = \psi_f(\tau) + f(\tau) + g(\tau)\, e^{-(2\tau/\kappa_p)} \quad (34)$$

and

$$\theta_f(1, \tau) = \psi_f(\tau) + f(\tau - \kappa_d\gamma)\, e^{-\gamma\kappa_d} + g(\tau - \kappa_d\gamma)\, e^{-(2\tau/\kappa_p)}\, e^{2\kappa_d(2-\gamma)/\kappa_p}\, e^{\gamma\kappa_d}. \quad (35)$$

Rewriting (35) in such a way that the arguments of f and g of (35) obtain the same values as in (34), f and g can be derived. With these expressions, the final approximate solution results, thus

$$\begin{aligned}\theta_f(\xi, \tau) = \psi_f(\tau) - [&\{e^{-\gamma\kappa_d(1-\xi)}\, e^{-(4\kappa_d/\kappa_p)(\gamma-1)\xi} \\ &- e^{\gamma\kappa_d(1-\xi)}\, e^{-4(\kappa_d/\kappa_p)(\gamma-1)}\}\{\psi_f(\tau - \kappa_d\gamma\xi) \\ &- \theta_f(0, \tau - \kappa_d\gamma\xi)\} + \{e^{-\gamma\kappa_d\xi} - e^{\gamma\kappa_d\xi} \\ &\times e^{-(4\kappa_d/\kappa_p)(\gamma-1)\xi}\}\{\theta_f(1, \tau + \kappa_d\gamma(1-\xi)) \\ &- \psi_f(\tau + \kappa_d\gamma(1-\xi))\}]/[e^{-\gamma\kappa_d} - e^{\gamma\kappa_d} \\ &\times e^{-(4\kappa_d/\kappa_p)(\gamma-1)}]. \end{aligned} \quad (36)$$

The solution shows that the temperature of a fluid element within the collector, $\theta_f(\xi, \tau)$, can be expressed in both its temperature at the moment of inlet, $\theta_f(0, \tau - \kappa_d\gamma\xi)$, and its temperature at the moment of outlet, $\theta_f(1, \tau + \kappa_d\gamma(1-\xi))$. During the time that the element concerned flows through the collector, the insolation and the heat losses, expressed in ψ_f, have their influences.

5. THE APPROXIMATE SOLUTION DISCUSSED

It is instructive to study θ_f, (36), for the limiting case of κ_p approaching zero. We notice the singular behaviour of $\theta_f(\xi, \tau)$ near $\xi = 0$, as a result of the factor $\exp(-4\kappa_d(\gamma-1)\xi/\kappa_p)$. This factor tends to zero for $\xi \neq 0$, but is equal to 1 for $\xi = 0$. Figure 3 qualitatively illustrates the behaviour of θ_f, (36), for various small values of κ_p.

An interesting result is found from (36), for $\xi \neq 0$ in the limiting case, $\kappa_p = 0$, giving

$$\lim_{\kappa_p \to 0} \theta_f(\xi, \tau) = \psi_f(\tau) + e^{\gamma\kappa_d(1-\xi)} \times \{\theta_f(1, \tau + \kappa_d\gamma(1-\xi)) - \psi_f(\tau + \kappa_d\gamma(1-\xi))\}. \quad (37)$$

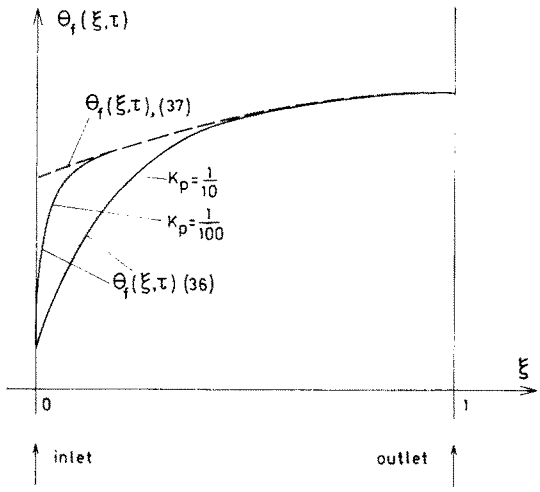

FIG. 3. The qualitative fluid temperature within the collector, $\theta_f(\xi, \tau)$, (36), for some small values of κ_p compared with the approximation $\theta_f(\xi, \tau)$, (37).

It appears that (37) is also the solution of the differential equation with $\kappa_p = 0$, namely

$$\gamma \frac{\partial \theta_f}{\partial \tau} + \frac{1}{\kappa_d} \frac{\partial \theta_f}{\partial \xi} + \gamma \theta_f = h(\tau), \tag{38}$$

satisfying the outlet temperature as a boundary condition. Referring to Fig. 3, we note that the right-hand side of (37) gives an excellent approximation of θ_f for nearly the whole region of ξ. Only in a boundary layer near $\xi = 0$ with a fast varying temperature, the approximation is rather poor. Practically, properties of interest mostly depend on expressions containing the temperature in integrated form with respect to the ξ-variable. For example, measurement or calculation of the mean value

$$\bar{\theta}_f(\tau) = \int_0^1 \theta_f(\xi, \tau)\, \mathrm{d}\xi \tag{39}$$

suffices in many cases. It will be clear that in those cases the integrated expressions are hardly influenced by the effect in the boundary layer for small values of κ_p.

We conclude that in the most practical applications of a conventional flat-plate solar collector, the right-hand side of (37) is a sufficient approximation for the fluid temperature. The expression is obtained by solving (38) using only the fluid outlet temperature as a boundary condition.

Expressed in the original coordinates, (38) transforms into

$$(C_p + C_f) \frac{\partial T_f}{\partial t} + \dot{m} c L \frac{\partial T_f}{\partial x} + \frac{T_f}{R_L} = \frac{T_a}{R_L} + \beta E(t). \tag{40}$$

It shows that in the first approximation the transient behaviour of the fluid temperature is completely described by one capacity consisting of the sum of the absorber and fluid capacities.

6. A DISCUSSION OF THE APPROXIMATE SOLUTION OF THE COMPLETE PROBLEM

In section 4 we derived that the approximate solution (36) of the second-order equation (16) is completely determined by the two boundary conditions of the inlet and outlet fluid temperatures, respectively. In order to describe the solution of the third-order equation (15), a third boundary condition has to be known. For example, the measurement of the time-dependent behaviour of the absorber temperature near the outlet, $\theta_p(1, \tau)$, would be convenient. With $\theta_f(1, \tau)$ and using (3a) the derivative, with respect to ξ, of the fluid outlet temperature could be defined as a third boundary condition, in this case.

An approximate solution of (15) for small values of κ_p and κ_g by similar methods as given in section 4 for solving (16) for small values of κ_p, is more complicated and will not be discussed here. We only assume that with respect to κ_p and κ_g the same conclusions can be made as at the end of section 5. We repeat that in most practical applications the fluid temperature within the collector can be approximated by solving (38) using the fluid outlet temperature as a boundary condition.

Finally, the temperature of the absorber, $\theta_p(\xi, 0)$ and the cover-plate temperature, $\theta_g(\xi, 0)$, within the collector, occurring in (5), can be approximated in first-order using (3a) and (1a), respectively, as

$$\theta_p(\xi, 0) = \theta_f(\xi, 0) \tag{41}$$

and

$$\theta_g(\xi, 0) = \theta_a(0). \tag{42}$$

7. APPLICATIONS

Performance calculations or test methods for solar collectors under transient conditions require the solution of a set of differential equations, such as (1–4), representing the mathematical model of the collector. Simple integration or Fourier transformation are the usual techniques to solve these time-dependent equations. In the solutions, the temperatures within the collector representing the heat stored capacitively, have to be known at both the beginning and the end of the measurements. Here we can use the approximations for the temperatures derived in the preceding sections.

For the sake of clearness we will shortly derive the relations of interest. Instead of considering the complete set (1–4), we limit ourselves to the simple equation (40). The solution will show us the character of the relations. At the same time, extensive mathematical calculations are avoided. Defining the finite Fourier transform for the time interval $0 < t < t_e$, as

$$\hat{T}_f(x, \omega) = \int_0^{t_e} \mathrm{e}^{-\mathrm{i}\omega t}\, T_f(x, t)\, \mathrm{d}t, \tag{43}$$

(40) transforms by the usual method, (see e.g. [5]), into

$$(1 + \mathrm{i}\omega R_L(C_p + C_f))\hat{T}_f + R_L L \dot{m} c \frac{\partial \hat{T}_f}{\partial x} = V(x, \omega), \tag{44}$$

with

$$V(x,\omega) = \hat{T}_a + \beta R_L \hat{E} - R_L(C_p + C_f) \times \{e^{-i\omega t_e} T_f(x,t_e) - T_f(x,0)\}. \quad (45)$$

Direct integration with respect to x gives the solution

$$\hat{T}_f(x,\omega) = \hat{T}_f(0,\omega)\, e^{-(1+i\omega R_L(C_p+C_f))x/R_L L\dot{m}c} + \int_0^x e^{-(1+i\omega R_L(C_p+C_f))(L-v)/R_L L\dot{m}c}\, V(v,\omega)\, dv. \quad (46)$$

Both the collector performance and a test method for collectors under transient weather conditions follow from the expression for the useful energy which is proportional to $\hat{T}_f(L,\omega) - \hat{T}_f(0,\omega)$. Using (46), a relation between the useful energy, the total insolation and the total heat losses can be derived of the type

$$Z(\omega) = \beta X(\omega) - Y(\omega)/R_L \quad (47)$$

where

$$X(\omega) = \hat{E}(\omega) \quad (48)$$

$$Y(\omega) = \hat{T}_f(0,\omega) - \hat{T}_a(\omega) \quad (49)$$

and

$$Z(\omega) = \dot{m}c(\hat{T}_f(L,\omega) - \hat{T}_f(0,\omega)) + i\omega(C_p + C_f)\hat{T}_f(0,\omega) + \frac{(C_p + C_f)}{L} \int_0^L e^{-(1+i\omega R_L(C_p+C_f))(L-v)/R_L L\dot{m}c} \times \{e^{-i\omega t_e} T_f v, t_e) - T_f(v,0)\}\, dv. \quad (50)$$

The relation (47) can be used as a test method to determine the collector parameters β and $1/R_L$. From measurements of the insolation, the ambient temperature, the mass flow and the inlet and outlet fluid temperatures, for various values of ω equations are obtained, yielding values for β and R_L if $(C_p + C_f)$ is known.

The temperatures inside the collector, $T_f(x,0)$ and $T_f(x,t_e)$, respectively, occurring in the integrand of the last term of (50) can be calculated only using the fluid outlet temperature as shown in section 5.

In a separate paper, more details of the test method will be given.

Particularly, for $\omega = 0$, (47) transforms into a relation, from which the collector performance can be derived, namely

$$\dot{m}c \int_0^{t_e} (T_f(L,t) - T_f(0,t))\, dt + \frac{(C_p + C_f)}{L} \times \int_0^L e^{-(L-v)/R_L L\dot{m}c} \{T_f(v,t_e) - T_f(v,0)\}\, dv = \beta \int_0^{t_e} E(t)\, dt - \frac{1}{R_L} \int_0^{t_e} (T_f(0,t) - T_a(t))\, dt. \quad (51)$$

In the literature, relations very similar to (51) serve as basic equations for transient test-procedures [7, 8]. If the collector has been described by a more extended mathematical model than (40), such as (1)–(4), again a relation of the type (47) can be obtained, but the functions corresponding to (48)–(50) are more complicated.

However, in that case, the temperatures inside the collector representing the heat stored capacitively can be calculated using the approximations (37), (41) and (42).

8. CONCLUSIONS

Calculations on a transient collector model require the collector temperatures within the collector to be known as initial conditions. In the conventional flat-plate solar collectors, with the exception of some boundary layers, these temperatures can be approximated using the fluid temperature solved from a single-capacity model with the measured fluid outlet temperature as a boundary condition. The approximations can be used to give a good approximation of quantities such as the heat stored capacitively, which are defined by expressions depending on the temperatures in integrated form.

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LES TEMPERATURES APPROCHEES DANS UN COLLECTEUR SOLAIRE PLAN SOUS DES CONDITIONS VARIABLES

Résumé—Quelques résistances thermiques de collecteur solaire plan conventionnel sont plus faibles que d'autres. Utilisant cette propriété, on obtient des approximations analytiques des températures dans le collecteur à l'instant $t = 0$. Elles sont exprimées en fonction du comportement variable dans le temps de l'isolation, de la température ambiante et des températures d'entrée et de sortie du fluide. On montre en particulier la variation rapide de la température du fluide près de l'entrée du collecteur. On mentionne quelques applications dans lesquelles des approximations plus grossières des températures sont possibles. Dans ce cas, une équation différentielle avec une capacité simple décrit la température du fluide.

NÄHERUNGSWEISE BESTIMMUNG DER TEMPERATUREN IN EINEM SONNENFLACHKOLLEKTOR UNTER INSTATIONÄREN BEDINGUNGEN

Zusammenfassung—Es gibt Wärmetransport-Widerstände an einem konvektionellen Sonnenflachkollektor, die kleiner sind als die übrigen. Damit läßt sich eine analytische Näherung für die Temperaturen im Kollektor zur Zeit $t = 0$ angeben. Sie werden als Funktion des zeitlich veränderlichen Verhaltens der Wärmedämmung, der Umgebungstemperatur und der Fluid-Austritts- und -Eintrittstemperatur angegeben. Insbesondere hat sich gezeigt, wie schnell sich die Fluidtemperatur nahe der Eintrittsstelle ändert. Einige Anwendungen werden erwähnt, bei denen gröbere Näherungen für die Temperaturen möglich sind. In diesem Fall läßt sich die Fluidtemperatur mit einer einfachen Differentialgleichung mit einer Wärmekapazität beschreiben.

ПРИБЛИЖЕННОЕ ОПРЕДЕДЕНИЕ ТЕМПЕРАТУР ВНУТРИ ПЛОСКОПЛАСТИНЧАТОГО СОЛНЕЧНОГО КОЛЛЕКТОРА В НЕСТАЦИОНАРНЫХ УСЛОВИЯХ

Аннотация—На некоторых участках обычного плоскопластинчатого солнечного коллектора тепловое сопротивление оказывается меньшим, чем на других. Используя это свойство, выведены аналитические аппроксимации для температур внутри коллектора в начальный момент времени $t = 0$. Они выражены через изменяющиеся во времени характеристики изоляции, температуры окружающей среды и температур жидкости на выходе и входе. В частности, отмечено быстро изменяющаяся температура жидкости вблизи входа в коллектор. Показан ряд случаев, когда возможны более грубые приближения для температур. В этом случае температуру жидкости можно описывать простым дифференциальным уравнением, в которое входит одно значение теплоемкости.